Converting Standard Form to Vertex Form

It is possible to change the equation of a quadratic in standard form $y = ax^2 + bx + c$ into vertex form $y = a(x-h)^2 + k$ using a method called completing the square.

Consider the vertex form of a quadratic relation: $y = a(x - h)^2 + k$

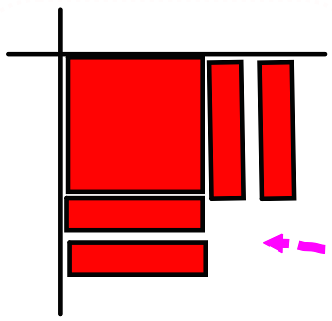
Vertex form is made up of ("a") groups of perfect squares with side length (x - h) and a constant term ("k") added. So, if we want to convert to vertex form, we must create (or "complete") a perfect square first.

Recall: $(x + \_\_)^2 = x^2 + 2(\_\_)x + (\_\_)^2$ or $(x - \_\_)^2 = x^2 - 2(\_\_)x + (\_\_)^2$

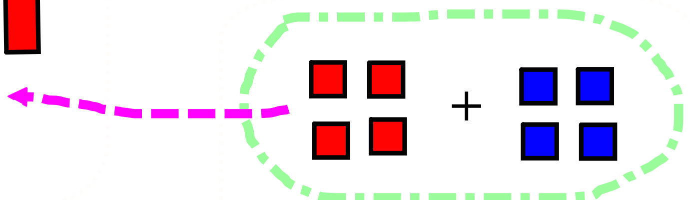
Example # 1: Write $y = x^2 + 4x + 7$ in vertex form.

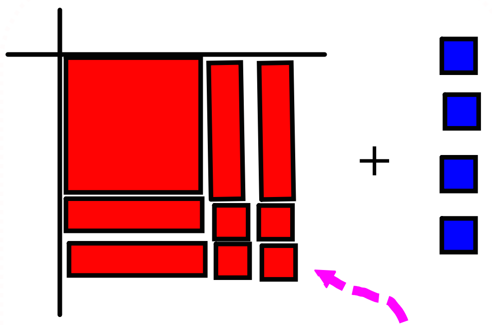
Vertex form requires a perfect square; focus on creating that square. Ignore the constant term 7 for now.

Use an area diagram. Place the x^2 first; in order for us to make a square, the bx term must be broken up evenly; half on each side.



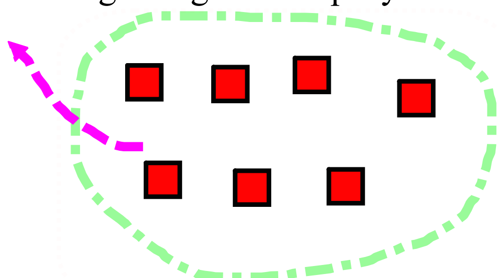
To complete the square, we need 4 unit tiles. To do this, we add 4 zero pairs!





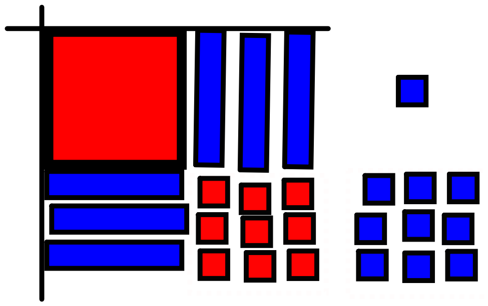
Note: It will always be positive tiles needed to complete the square!!

To finish, add the seven unit tiles we ignored in the beginning and simplify!



$$x^2 + 4x + 7 = (x+2)^2 + 3$$

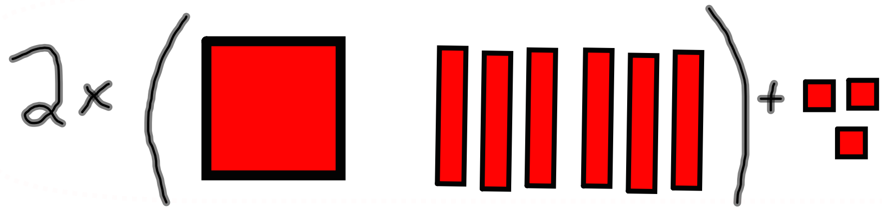
Example # 2: Write in vertex form: $y = x^2 - 6x - 1$



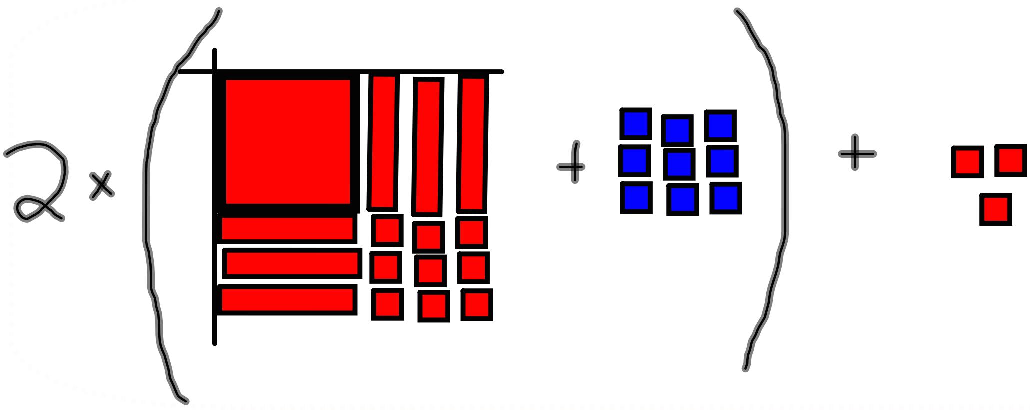
$$y = x^2 - 6x - 1$$
$$y = (x-3)^2 - 10$$

Example # 3: Write in vertex form: $y = 2x^2 + 12x + 3$

When there is a numerical coefficient of x^2 other than 1 it is our "a" value or stretch factor. We must divide the x^2 and x tiles into "a" equal groups to work with. Complete the square in one group. We know then that we have "a" of the squares, as well as "a" times the number of tiles left over from completing the squares.



Next step: Complete the square!



$$y = 2x^2 + 12x + 3$$
$$y = 2(x^2 + 6x) + 3$$
$$y = 2(x^2 + 6x + 9 - 9) + 3$$
$$y = 2(x^2 + 6x + 9) + 2(-9) + 3$$
$$y = 2(x+3)^2 - 18 + 3$$
$$y = 2(x+3)^2 - 15$$

Formal solution should look like this; your work with the tiles goes on the side as rough work!

Algebraic Method:

Example # 4: Change $y = 5x^2 + 20x + 2$ into vertex form by completing the square.

Steps:

- common factor the numerical coefficient that is in front of the x^2 term out of the first two terms only. Ignore the constant term at the end.
- create a perfect square trinomial inside the bracket by adding and subtracting the square of half the numerical coefficient in front of the x-term.
- using only the first three terms, change the perfect square trinomial to a binomial squared.
- remember that the fourth term inside the bracket must be affected by the common factor that you originally removed before it can be collected with our constant term that we have been ignoring.

$$y = 5x^2 + 20x + 2$$
$$y = \left(5x^2 + 20x\right) + 2$$
$$y = 5\left(x^2 + 4x\right) + 2$$
$$y = 5\left[x^2 + 4x + (2)^2 - (2)^2\right] + 2$$
$$y = 5(x+2)^2 + 5(-4) + 2$$
$$y = 5(x+2)^2 - 18$$

Example # 5: Change $y = -2x^2 + 12x - 7$ into vertex form by completing the square.

$$y = -2x^2 + 12x - 7$$
$$y = -2\left(x^2 - 6x\right) - 7$$
$$y = -2\left[x^2 - 6x + (3)^2 - (3)^2\right] - 7$$
$$y = -2(x-3)^2 - 2(-9) - 7$$
$$y = -2(x-3)^2 + 18 - 7$$
$$y = -2(x-3)^2 + 11$$

Homework: p. 390 # 4 , 5 , 9ab